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RESEARCH ARTICLE

The Bell Inequality and Nonlocal Causality

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Abstract—The Freedman–Clauser experiment in Berkeley and the Aspect experiment in Orsay were the defining physical experiments demonstrating nonlocal causality in quantum mechanics. They each counted coincidence measurements on entangled polarized photons from a common source. This article begins with a brief discussion of the quantum mechanics of polarized photons. We show an example of the changes in the count rates when the polarizers are changed under assumptions of local causality. This causes a contradiction with quantum mechanical predictions. The example uses a logical flow and the algebra of inequalities. It constitutes a conditional proof of the Bell inequality. Next we discuss the experimental background and the events leading up to it. We discuss several hypotheses in explanation, of which our favored is the time reversal of cause and effect.

Keywords: Bell's theorem—causality—entanglement—nonlocality—time reversal

Introduction

The goal of this article is to provide an accessible description of Bell's inequality and the physical basis of nonlocality. Bell's theorem (Bell 1964) is a mathematical inequality that proves that under some conditions quantum mechanics is inconsistent with local causality. What is local causality? Figure 1 is a spacetime diagram with time increasing horizontally to the right and the space dimensions represented vertically. It shows two correlated particles emerging from a common source, moving off in opposite directions and moving forward in time. Local causality implies that the cause of the particles' presence and properties is at the source. Nonlocal causality implies the cause occurs elsewhere. Imagine a baseball in flight. Is the cause of a baseball's flight with the batter, or with the glove that catches the fly ball? Food for thought.

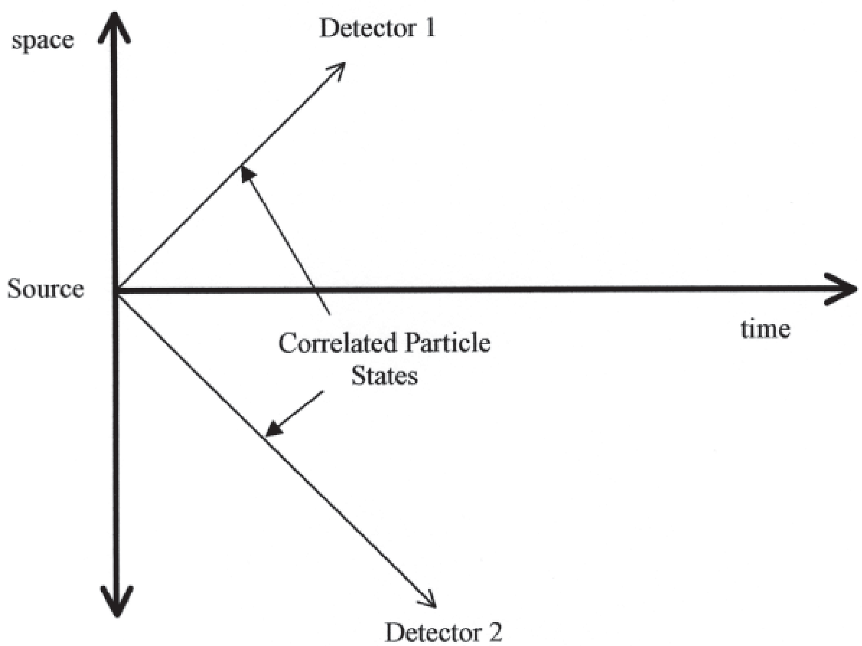

Figure 1. Space–time diagram of two entangled particles departing from a common source and moving apart in opposite directions in space and forward in time.

There are many demonstrations, or limited proofs, of Bell's theorem. The present treatment, adapted from Clauser, Horne, Shimony, and Holt (1969) (CHSH), is less abstract and more relevant to the experimental tests that have been performed. The CHSH abstractions have been mellowed by applying techniques used by Henry Stapp (1977)

Bell's theorem has been tested experimentally many times. Among the first were experiments performed by Freedman and Clauser at Berkeley (April 1972) and by Aspect, Grangier, and Roger in France (July 1982). Tests like these have shown that quantum mechanics rules and local causality fails.

We will give a limited demonstration of the theorem pertinent to an experiment with polarized photons. We show a nutshell version of Bell's theorem and its background. We show the derivation of the probabilities for quantum mechanical polarization states for two-photon emission, followed by a conceptual entangled particles experiment using two polarized photons emitted from an excited atom, much the same as was done in the original experimental tests. We show the quantum probabilities for measurement outcomes of the photon states, and discuss the local causality constraints on the outcomes. We review some of the early experimental evidence. Finally we give some philosophical implications of nonlocal causality. Among several hypotheses, that of causal time reversal emerges as the most cogent explanation for nonlocal causality.

A Few Definitions

We need to define some terms before we launch into the analysis.

▪ LOCAL CAUSALITY—This occurs when effect follows cause, staying within its light cone, and is contiguous to it in spacetime. This is causality as normally experienced, with which we are all familiar. It is a consequence of the assumption that elementary particles have an objective existence which persists throughout their spacetime paths.

▪ WAVE FUNCTION—(State function; State vector) A solution of the dynamical equations of quantum mechanics, describing the development in spacetime of the probability of a measurement outcome.

▪ SUPERPOSITION—A linear combination of wave functions, giving multiple possible measurement outcomes with different probabilities. Note that a superposition of states may have two interpretations. First is the existence of a single particle in a single undetermined dynamical state, the probability of which state is predicted by quantum mechanics. Or, it may imply that a distribution of potential states exists, only one of which is "born" when a measurement is made.

▪ WAVE FUNCTION COLLAPSE—A measurement process will result in one of multiple possible measurement outcomes in a superposition. The disappearance of the superposition, to be replaced by a single state, is called a collapse. The state is an eigenfunction, or a root function of the measurement.

▪ HIDDEN VARIABLES—Hypothetical dynamical variables of hidden particles, of which quantum mechanics gives only the probabilities. These are supposed to be real, but are not measurable with current technology.

▪ COPENHAGEN INTERPRETATION of quantum mechanics—The centerpiece of the philosophy of quantum mechanics, this interpretation says that there are no hidden variables. All measurements are probabilistic. An experimenter's choice of measurement will determine the form of the wave function as a solution of the dynamical equations. The interpretation is quite pragmatic. The Copenhagen interpretation was authored principally by Niels Bohr and Werner Heisenberg, and is so named because Bohr did his work at the Bohr Institute at the University of Copenhagen.

▪ ENTANGLED PARTICLES—These are widely separated particles in spacetime which belong to the same quantum state, or superposition of states, such that measurements on one of them are strongly correlated with measurements on the other. “Entanglement” is a term invented by Heisenberg to refer to the EPR paradox. Its use has been expanded to include other systems of the same genre.

▪ LORENTZ INVARIANCE—This refers to the invariance of the laws of physics under transformation from one relativistic uniformly moving reference frame to another.

The EPR Paradox and Bell’s Theorem

Anyone who understands Bell’s theorem
and isn’t bothered by it has got rocks in his head.

Albert Einstein, the principle author of Einstein, Podolsky, and Rosen (EPR) (1935) considered the Copenhagen interpretation of quantum mechanics to be incomplete. To illustrate his case, he and his colleagues proposed a *gedankenexperiment.*

In this experiment, entangled particles with correlated states are emitted from a source, going in opposite directions. They impinge on two measuring detectors. The situation is represented in Figure 1. Until they are measured, the states are a superposition of several possible outcomes. When a measurement is made at Detector One, the state at Detector Two immediately collapses into the correlated state. This is in spite of the fact that the detectors are widely separated, and that there is no time for a light signal to pass between them. Einstein characterized this as “spooky action at a distance,” and thought it to be impossible.

The EPR paradox is based on the assumption of local causality for the entangled particles. But Bell’s theorem and the resulting mathematical inequality shows that quantum mechanical predictions of measurement outcomes for certain entangled states are inconsistent with local causality. We will show an example of how this may come about, which constitutes a limited proof of nonlocal causality. The discussion is an adaptation of Henry Stapp’s treatment in *Il Nuovo Cimento* (1977).

Bell’s theorem applies to the quantum mechanics of atomic particles. It was published almost fifty years ago in the first issue of *Physics*, and the journal folded after that one issue. For the next decade, the Bell paper seemed to the scientific world to be underwhelming.

The mathematical inequality has been tested repeatedly, and testing shows that quantum mechanics rules. Local causality often fails.

Quantum Mechanical Polarization States

The experimental concept to be described below depends on a linearly polarized light beam. Correlated photons are emitted from a common source, each polarized so that the electric field vibrates in the same plane. The emitted light beam, and each photon in it, has a superposition of all polarization states. If a measurement is made with a vertical polarizer, the superposition collapses into a single state, a vertically polarized beam.

A measurement of the new state at an off-angle does not, however, result in a null state. If a singly polarized state is passed through another polarizer at an angle theta, much like a force, the polarizer will capture its projection on the polarizer axis.

The analogy is that the wave function of the light beam is equivalent to an electromagnetic field, and an electromagnetic field is a force field. The polarizer is free to vibrate along the polarization axis under the influence of the force, but vibrations across the axis are forbidden by the structure of the polarizer molecules.

A Conceptual Entangled Particles Experiment

The experimental setup shown in Figure 2 was used by Stapp in his demonstration, and is also representative of the first experiments done to prove that quantum mechanical predictions prevail and locality fails.

The source is a vapor of calcium ions in a heated oven or an atomic beam, energetically emitting two correlated photons in opposite directions. The emission is a "cascade" process—one photon is followed quickly by the second. They share the same plane of polarization, one up and one down. The direction of polarization is an indeterminate superposition until it is measured. The polarizer axes are set at an angle of theta, one to the other. This angle is an important part of the subsequent analysis.

Local causality dictates that the source ion causes a photon to be emitted in a given direction and polarization state. This state is a "hidden variable." Local causality requires hidden variables. The photon incident on the polarizer is either absorbed or causes a new photon to emerge from the polarizer. The emergent photon, if there is one, causes a signal in the detector.

Each detector records an apparently random sequence of events. The current wisdom says that no information is transferred faster than light speed. The coincidences are not detectable until the two data records are brought together through classical information channels, after which they

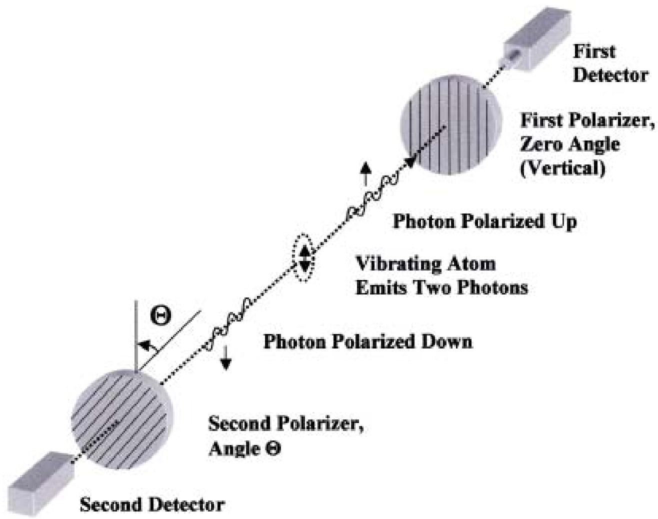

Figure 2. Experimental concept for coincidence measurement of entangled polarized photons.

are compared. Then the correlations are apparent.

Recent experiments described in *Physics Today* (April 2011) indicate that information transfer may be "enhanced" with the use of entangled systems, which increase the efficiency of a transmitting channel to nearly theoretical values. The concept of "no information transfer" needs further review, and a careful definition of the meaning of "information."

Quantum Probabilities for Measurement Outcomes

Figure 3 is a template we use to display the probabilities, or normalized measurement rates, of two coincident photons through their respective polarizers. The circles with arrows in them are icons representing the polarizer orientations. The front and rear polarizer angles are shown to the left of these circles. The angle Θ is the included angle between the two polarization axes. The four large boxes each have a pair of polarizer settings representing the front and rear polarizers. The chosen angles were shown by CHSH (1969) to give the greatest violation of the Bell inequality.

Each of the four large boxes contains four small boxes, labeled **(a)** through **(d)**. Each of these in turn contains two circles, again representing the front and rear polarizers. They represent the four possible outcomes

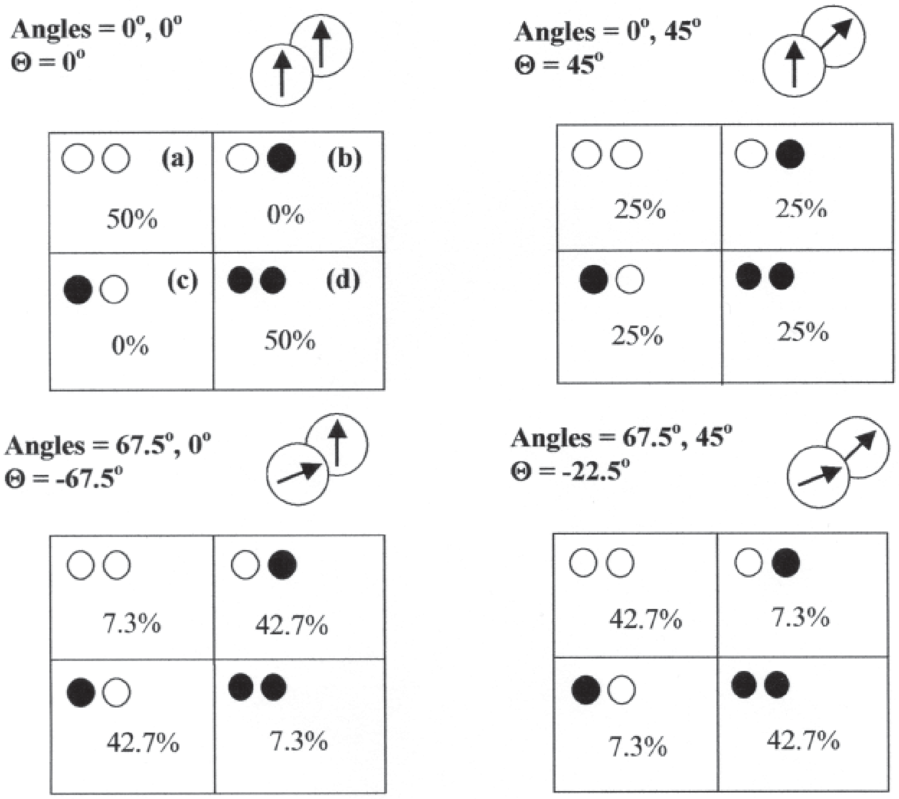

Figure 3. Template showing quantum mechanical measurement probabilities for four different polarizer settings and four different photon incidence event types.

of the photon measurements. The open circle represents transmission of a photon through the polarizer. The shaded circle represents absorption, or no transmission. Event **(a)** is a coincidence measurement. Event **(d)** is no measurement (null event). The other two are mixed transmission and absorption events which we will call anti-coincidences.

Figure 3 shows the measurement probabilities for each of the sixteen displayed states as predicted by quantum mechanics. The formulae used are $\frac{1}{2}\cos^2\Theta$ in small boxes **(a)** and **(d)**, and $\frac{1}{2}\sin^2\Theta$ in small boxes **(b)** and **(c)**. Notice that if the polarizers are parallel, every event appears either as a coincidence (open–open, 50%) or as an absorption of both photons (black–black, 50%). There are no anti-coincidence events.

The formulae may be derived using an incident quantum mechanical wave function, $e^{i\varphi}\sqrt{(2\pi)}$, with φ equal to the superposition of incident photon polarization angles. The expectation value of a complex coincidence operator, a function of the polarizer angle Θ, is involved.

A more intuitive approach may be taken. Suppose two incident photons succeed in passing through the polarizers. Then one or the other of the polarizers, with axis at an angle Θ, will have the photon's field strength reduced to its projection on the polarization axis. This gives a factor of cos Θ. This is a consequence of the force-like nature of the field strength, described in the above section on polarization states. Quantum mechanics tells us that the probability of seeing a measurement is proportional to the square of the field strength (or wave function) of the measurement. So the probability of seeing a coincidence in small box **(a)** is proportional to $\cos^2 \Theta$.

A careful analysis shows that the probability of a uniform superposition of polarizations passing a photon through a polarizer is one-half. That is, on average, half the photons are sufficiently aligned with the polarizer axis to get through and half are not. Consequently the proportionality factor of the coincidence measurement is ½, and the probability is $\frac{1}{2} \cos^2 \Theta$.

Suppose one polarizer transmits and the other absorbs. The probability of this anti-coincidence event together with a coincidence event is one-half. So the anti-coincidence must have a probability of $\frac{1}{2} \sin^2 \Theta$. This formula calculates probabilities for small boxes **(b)** and **(c)**. Conservation of probability then dictates the probability of the null event to be $\frac{1}{2} \cos^2 \Theta$. Then the sum of the four probabilities equals one for all polarizer angles.

Local Causality Constraints on the Outcomes

The squares of the trig functions derived in the last section will be used to calculate the quantum mechanical coincidence rates. These will be compared to local causality constraints, under the assumption that local causality is compatible with quantum mechanical predictions. Contradictions of these constraints with quantum mechanics will demonstrate that they are not compatible.

The procedure to follow will change each one of the polarizer settings sequentially to move from the upper left to the lower right polarizer settings (large boxes). As we do so, we examine the constraints of local causality for the changes in each coincidence rate (small boxes). The procedure is illustrated in Figure 4.

To facilitate the math, we need to define a fraction f, given in percent, of the total number of all measurements being moved along each of the two paths. This fraction would appear as the percentage of measured coincidences (open–open) at polarizer angle zero–zero which gets absorbed (black–black) as a result of changing the polarizer settings to 67.5 and 45

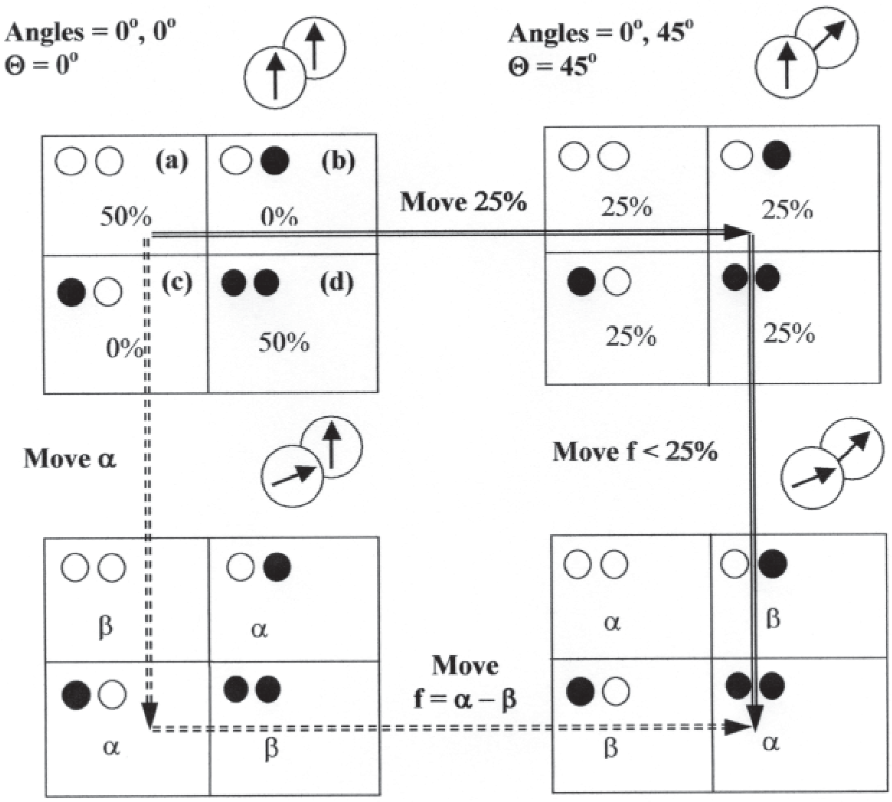

Figure 4. Template for analysis of local causality constraints; a fraction f moves from event type (a) to event type (d).

degrees. The solid and dashed arrows represent two different sequences in which the polarizer angles are changed. The solid sequence changes the rear polarizer first, the dashed sequence changes the front polarizer first.

In Figure 4, follow the sequence of the solid arrows. To begin, only the second (rear) polarizer changes orientation. Imagine that for each polarizer setting the exact same set of photons emerges from the source and is incident on the polarizers. As we change the polarizer setting, we see how events are changed from coincidence events to anti-coincidence events. The key is that the locally caused photons are like real objects. Photons incident on the unchanged polarizer will transmit exactly as they did before. Photons incident on the changed polarizer will be partially blocked due to the polarization misalignment. Local causality allows measurement events in **(a)** to be converted only to events in **(b)**. Thus far, everything is assumed to be compatible with quantum mechanical coincidence rates. So the fraction of measurements moving from coincidence to anti-coincidence, by the ½ $\sin^2 \Theta$ formula, is 25%.

Next, change the front polarizer to a new angle of 67.5 degrees. Then local causality will allow changes only in the transmission rates in the front polarizer, and measurements can move only from small box **(b)** to small box **(d)**. The fraction of measurements f moving from coincidence to null (pass–pass to block–block) is 25% at most.

Now we are going to have locality constraints on the incidence rates appearing in the two lower large boxes, and the results are unknown. Accordingly, we change the numbers to algebraic symbols. But we still assume compatibility with quantum mechanics. Compatibility requires that α and β trade places between the lower two large boxes in Figure 4. Conservation of probability also requires that α and β add up to 50%. Following the dashed arrows, we first change the front polarizer to an angle of 67.5 degrees. Due to local causality, the same photons will still be passed by the rear polarizer, but some of the coincident photons on the front polarizer will now be blocked.

Local causality allows the coincidence measurement events in **(a)** to only be converted to anti-coincidences in **(c)**. All of the α events now in **(c)** must come from **(a)**. Next, change the rear polarizer to a new angle of 45 degrees. Then local causality will allow changes only in the number of events detected in the rear polarizer, and measurements can move only from small box **(c)** to small box **(d)**. The fraction of measurements moving is $\alpha - \beta$, since β of them must remain in small box **(c)**.

Note that the solid and dashed path protocols may be interchanged, obtaining the same results.

Two of the constraints on f are as shown in Figure 4. The third constraint on $\alpha + \beta$ derives from the symmetry of α and β and conservation of probability, that all incidence rates must add up to 100 percent. In summary, the constraints are:

$$f < 25 \qquad f = \alpha - \beta \qquad \alpha + \beta = 50$$

Eliminating f from the three equations gives a solution for α and β.

$$\alpha < 38 \qquad \beta > 12$$

This appears as an inequality constraint, and is a special case of the Bell inequality. These two constraints now apply in the bottom two large boxes of Figure 4, where the front polarizer orientation is at 67.5 degrees.

Comparing these with the quantum mechanical predictions for the same polarizer settings shown in Figure 3, it is apparent that the quantum mechanical predictions violate the local causality constraints.

This demonstrates the incompatibility of local causality with quantum mechanical predictions for entangled, polarized photons, under the particular polarizer conditions chosen.

We may ask why we can't write an exact equality for the fraction f rather than the inequality we have shown above. This would be a strong constraint on the compatibility of quantum mechanics with local causality, rather than the weak (inequality) constraint shown. We could do this. We have in fact imposed a strong constraint by demanding in Figure 4 that the measurement probabilities for quantum mechanics and local causality be equal for parallel polarizer axes (upper left large box). But local causality could be compatible with quantum mechanics even within a range of measurement probabilities. So we have weakened the constraint in passing to the large box on the right. We did this with the intention of solving for the range of compatibility.

Experimental Evidence for Nonlocality

Many experimental tests have been done to validate quantum mechanics and to demonstrate that entanglement leads to nonlocality of cause and effect.

These tests give rise to considerable tension between two sacrosanct theories, quantum mechanics and local causality. These are two of the finest of fundamental physics. They are at odds with each other and will not coexist. Which will the test prove to be correct? From the beginning, there was really no question. Nobody beats quantum mechanics.

The landmark tests were done with cascaded polarized photons from ionized calcium vapor. The first test, the breakthrough, was the Clauser–Freedman experiment (1972) done at Berkeley.

There were a number of criticisms of the experimental controls, the lack of which might have provided an (unlikely) opportunity for misinterpretation. The results were statistically significant.

Aspect performed what is now considered the defining test, with tighter experimental controls, at the University of Paris in Orsay. His experiment included changing the polarizer settings while the photons were on the fly. With passing years, the tendency is to refer only to the Aspect experiment and, perhaps unfairly, to overlook the original Clauser–Freedman experiment.

Philosophy of Nonlocal Causality

Are we not drawn onward, we few, drawn onward to a new era?

If the cause-and-effect does not begin at the source, with the emitting ions, where is it? Is the photon caused by the observation? Is the cause at the detector? This explanation is problematic. But perhaps we are misled, as we are by the palindrome, into thinking time has only one direction. However, let us first consider a few of the more popular alternatives.

A common explanation, perhaps the most widely used, is that entangled particles form a single quantum state, and will respond to a measurement as any state does, by yielding a sub-state out of the superposition. This is pure, pragmatic Copenhagen interpretation, and really begs the question of the structure of cause and effect.

One might argue that causality as a physics principle is outside the realm of spacetime. Then its true structure remains to be discovered, as does its mode of interaction with spacetime. Another argument holds that causality is a human-derived artifact, an appearance of a larger reality, useful only in describing our experience of spacetime.

On the other hand, if there are hidden causal interactions between the two detectors and the source, such as David Bohm's pilot wave, their responses may be guided as specified by quantum mechanics.

In a variant of his experiment, Alain Aspect (Aspect, Dalibard, & Roger December 1982) modified his apparatus so that the polarization of Detector One was set after the photons had left the source. The polarization change was accomplished in a pseudo-random fashion, so it is difficult to imagine how hidden causal interactions could govern the final detector dynamics before the source emitted its photons.

The tentative conclusion from the hidden causal interactions hypothesis is that if there is a hidden transmission of causal interaction from one detector to the other it must be "superluminal" (faster than light speed). In Figure 1, this would correspond to an arrow pointing from one detector to the other. But there is one other possibility.

Figure 1 shows arrows pointing away from the source, indicating a timewise forward motion of the particles, and implying a causal link forward in time. If one of the arrows pointed back toward the source, this could be taken as a causal link starting at the detector, which more or less randomly collapses the wave function into a single polarization state, which then collapses the source into that state in a retrocausal fashion. The unique state thus assumed by the source will then cause the measurement at the other detector to be suitably correlated. This is time reversal of cause and effect.

The symmetry of dynamic time reversal is well-known in field theory. There are no theoretical reasons that would rule out causal time reversal if dynamic time reversal is possible.

Causal time reversal is attractive because it maintains contiguity through the photonic paths, and the process remains Lorentz-invariant, requiring only that we use improper Lorentz transformations where the time element of the metric is negative instead of positive.

The problem is, causal time reversal is anathema to many physicists. It is way outside the paradigm, it seems, and many cannot grasp the concept. The widespread concept is that cause precedes effect by simple definition. Our life experiences have left us with neural patterns that preclude the opposite concept.

If we accept causal time reversal, we must ask why the primary cause should be at Detector One, and not at the more distant Detector Two. Perhaps both detectors contribute to causality in ways dependent on the experimental/observational setup.

No one has thought of a good objective physical test that will prove or disprove causal time reversal. But causal time reversal may emerge on the macroscopic level, especially when certain types of cognitive action comes into play. Causal time reversal may explain some types of cognitive action yielding foresight or premonitory effects. Dean Radin (1997) has done experiments to show presentiments of startling events using galvanic skin reaction measurements. Dobyns (2006, AAAS San Diego meeting) has shown that certain types of premonitions or precognitive processes require causal time reversal to explain them. Some other types of rare processes involving apparent entropy decrease in evolving systems also seem to fit a causal time reversal paradigm.

In retrospect, nonlocal events may nearly always be accompanied by time reversal of cause and effect. They are, in a way, synonymous with each other.

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